Modelling bank leverage and financial fragility under the new minimum leverage ratio of Basel III regulation[1]

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Abstract

We analyse the determinants of banks' balance sheet and leverage ratio dynamics, and its role in increasing financial fragility. Our results are twofold. First, we show there exists a value of bank leverage minimising financial fragility. Second, this value depends on the overall business climate and the expected value of the collateral provided by firms. Based on our findings, we argue that an adjustable leverage ratio restriction dependent on economic conditions would be preferable to the fixed ratio included in the new Basel III regulation.

Keywords: Bank leverage; leverage ratio; financial fragility; prudential regulation.

1. Introduction

The devastating consequences of the 2008 financial crisis in terms of economic activity and unemployment have reignited debate on the causes of financial fragility and instability. Allen and Carletti (2010), Brunnermeier (2009), Greenlaw et al. (2008) provide empirical overviews of these events and the financial crisis. The financial crisis has been associated to a number of factors such as the housing and credit markets. Suggested causes include the inability of homeowners to make their mortgage payments, overbuilding during the boom period, high personal and corporate debt levels, financial product innovation, failure of key financial institutions, and errors of judgment by credit rating agencies in the rating of structured

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products. Macroeconomic factors such as accommodating monetary policy, global imbalances, and government regulation (or lack thereof) are also considered to have played a direct or indirect role in the crisis.

Another factor that has been highlighted is the significant increase in bank leverage levels in the four or five years preceding the crisis that broke in summer 2007 and the panic in autumn 2008, particularly amongst the major European banks and US investment banks. The banks leverage increase was around 50% in some cases. The level of asset-to-equity ratios (the *equity multiplier*) remained close the 20-25 range. This implied a capital-asset ratios (or *leverage ratio*) of 5% to 4%, until 2003-2004, with significant heterogeneity across regions and categories of banks.[5] Between 2005 and the onset of the crisis, the top 50 major global banks amongst US investment banks and European universal banks had an equity multiplier close to or even exceeding 30. Therefore, their leverage ratio was only 3% (Financial Stability Forum and Committee on the Global Financial System Joint Working Group, 2009).

This excess leverage prior to the crisis and the devastating impact of the deleveraging in its wake convinced the G20 and the prudential supervisors that a leverage ratio restriction should be added to the traditional prudential measures. It was envisaged that this would be complementary to the prudential risk ratios and would therefore not replace the Basel II or Basel III micro-prudential regulation elaborated at the time (Ingves, 2014). This leverage ratio is a measure of a bank's Tier 1 capital as a percentage of its assets plus off-balance sheet exposures and derivatives. The Basel Committee chose a minimum leverage ratio of 3% and a maximum equity multiplier of 33. The implementation of this ratio on an experimental basis began in January 2013. After various adjustment phases between 2015 and 2017, the leverage ratio became imperative in Pillar I of Basel III in January 2018.

However, the effectiveness of such a regulatory leverage ratio restriction is questionable. If the chosen value of the ratio is too low, it will have a detrimental impact on the banks' ability to provide loans. If the chosen value of the ratio is too high, it will not prevent banks' excessive risk taking. In both cases, the question of an existing leverage ratio value minimising the likelihood of bank bankruptcy (hereafter referred as banks' financial

5. In line with the terminology used in Basel III, 'equity multiplier' refers to the 'asset-to-equity ratio' and 'leverage ratio' refers to the 'capital-asset ratio'.

fragility) needs to be addressed jointly with credit availability. This is the objective of this paper.

We develop a model of financial intermediation where leverage and the pricing of bank assets (interest rate on loans) are endogenously determined and depend in equilibrium on the overall business climate. In this framework, we show that financial fragility can result from bank profit maximisation even though the micro-prudential requirements laid down by the Basel III Internal Rating-based (IRB) capital regulation are met. The model also allows us to investigate the existence of an "optimal" leverage ratio minimising financial fragility.

Our analysis provides two main results. First, we show that there is a non-linear relationship between the level of bank leverage and financial fragility, defined as the critical level of macroeconomic shock triggering bankruptcy. More precisely, we show there is an optimal value of leverage minimising financial fragility which allows for the identification of two states of the economy: the "inefficient equilibrium state" and the "trade-off equilibrium state". In the "inefficient equilibrium state", high levels of financial fragility are associated with low bank leverage and low levels of credit availability. In the "trade-off equilibrium state", high levels of financial fragility are associated with high bank leverage and high levels of credit availability. This result underlines that bank leverage can increase without being detrimental to financial fragility, as long as the level of bank leverage is lower than the level minimising financial fragility. This result sheds light on the potential impacts of the new Basel III capital regulation, which introduces a maximum value for bank leverage. If the maximum value fixed by the regulatory authorities is too low, the economy can become trapped in the "inefficient equilibrium state", while excessively high maximum leverage will stimulate credit availability to the detriment of financial fragility. This result is in line with Kashyap and Stein (2004) who argue that a policy maker concerned about the objectives of both financial stability and maintaining credit creation must sometimes be willing to tolerate a higher probability of bank failure. Importantly, securitization could amplify the financial fragility mechanism presented in the paper. Securitization allows banks to increase their off-balance-sheet activities resulting in a rise in the global level of bank leverage and thus intensifying financial fragility. Nevertheless, we have decided not to consider this issue. Our objective is to show that financial fragility may be the outcome of an economic system without any informational problems between banks and the regulator or specific practices of risk reduction, like securitization.

Second, we show that both the bank's chosen equilibrium level of leverage and the value of leverage that minimises financial fragility, depend on the overall economic situation and on the expected value of the collateral provided by firms to the bank. We show that there is a critical threshold above which an increase in the expected value of collateral leads to an increase in financial fragility. This result is in line with the growing literature that explicitly relates bank behaviour, endogenous debt growth and financial instability (Schularick and Taylor (2012), Phelan (2016) or Galo and Thomas (2017)). Moreover, since the optimal level of leverage minimising financial fragility depends on the overall business climate, we advocate for the establishment of an adjustable leverage ratio that depends on the economic conditions, rather than the fixed ratio provided for under Basel III. This result is in accordance with Repullo and Saurina's (2011) argument that a proper assessment of bank risk should be conducted conditionally on the state of the economy, not unconditionally.[6]

Our contribution is related to two well-established strands in the literature: literature on bank leverage and financial fragility as well as literature related to leverage ratio restriction. Minsky (1982, 1986) are amongst the most important contributions adressing the question of bank leverage and financial fragility. These contributions develop a business cycle theory based on a financial conception of economic fluctuation and propose the "financial instability hypothesis". In Minsky's approach, banks' profit-seeking behaviour leads them to deliberately reduce their capital-asset ratio and engage in financial operations involving high leverage when their activities are expanding. Contemporary economists such as Goodhart (2010) and Roubini and Mihm (2010) have underlined that the recent financial crisis is based largely on similar mechanisms. Several contributions assign great importance to debt leverage in the dynamics of financial instability. Geanakoplos (2010a,b) postulates a leverage cycle as a recurrent phenomenon in US financial history. In a series of articles on the subprime crisis, Adrian and Shin (2010a,b) examine the role of financial intermediation in the 2007-2009 financial crisis as well as the role of leverage effects. They emphasize the pro-cyclicality of leverage and the positive relationship between leverage and the size of financial

6. Kashyap and Stein (2004) propose time-varying capital requirement as an optimal scheme for bank regulation.

intermediaries' balance sheets, especially before the crisis. Similarly, Shin (2009) models a lending boom fuelled by declining measured risk. He shows that in benign financial market conditions when risks are low, financial intermediaries expand their balance sheets as they increase leverage. Of course, there is a symmetrical process which accentuates the magnitude of the crisis when the risks are high, leading to sharp deleveraging, then a credit crunch.

With respect to this literature, our analysis has two contributions. First, we show that even under an "ideal" economic environment (perfect information, economic expansion, optimistic expectations, rising asset prices, rational agents within the standard meaning of the term), pro-cyclical financial fragility based on the relationship between asset prices and the bank's lending cycle, can appear. Second, we show that there is a non-linear relationship between leverage and financial fragility.

The need for leverage ratio restrictions has been studied by the literature. Schoenmaker (2013) reviews the historical developments and arguments that led to the recommendation of a mandatory leverage ratio by the Basel committee. However, theoretical works on this issue focus mainly on the disciplinary effect induced by a leverage ratio on the bank's risk loan declaration. In a seminal paper, Blum (2008) shows that banks can report their level of risk untruthfully in a Basel II framework. In this context, a risk-independent leverage ratio restriction may be necessary to induce truthful risk reporting. However, Blum does not propose an assessment of such a ratio value. Similarly, Kiema and Jokivuolle (2014) focus on the impact of a risk-independent leverage ratio restriction on model risk which arises if some loans are incorrectly rated by a bank in IRB approach of capital regulation. In contrast to Blum (2008), they show that such a leverage ratio restriction might induce banks with low-risk lending strategies to diversify their portfolios into high-risk loans, which could undermine banking sector stability. They show also that in order to overcome this negative effect, the risk-independent leverage ratio must be higher than the ratio required by Basel III regulation. Jarrow (2013a), which is closer to our view, tries to provide a rationale for determining the value of a maximal leverage ratio based on Value at Risk rules. In his contribution, this value depends on the bank's microeconomic characteristics and especially the structure of its balance sheet. In our model, the optimal leverage ratio value depends on the macroeconomics condition, and it is not exclusively related to the specific characteristics of one bank.[7]

The rest of the paper is organised as follows. Section 2 presents the model, Sections 3 and 4 present and discuss the main results. Section 5 concludes.

2. The model

We consider three classes of agents – firms, individual investors and a bank – and two periods. In the first period, firms need external funds in order to invest in a risky project subject to a macroeconomic shock. We assume that firms have access only to bank loans. In this period, financial contracts are signed between borrowers and the bank, and investment decisions are made. In the second period, the value of the macroeconomic shock and the effective return on investment are known. Non-defaulting firms have to pay for their external funds and defaulting firms are liquidated. Finally, in accordance with the literature (Dell'Ariccia, alii. 2014, Holmstrom and Tirole, 1997) all parties are risk-neutral and protected by limited liability.

2.1. Firm and bank behaviours

In period 1, firms with zero wealth have access to a risky investment project whose undertaking requires one unit of wealth. We assimilate firm and project and assume that firms (projects) are uniformly distributed over $[0,1]$ according to the level of their intrinsic or specific characteristics x_i, whose value is common knowledge to all the agents in the economy. There is no financial market, and since firms lack capital, they need to borrow the total amount of their investment from a bank.

In period 2, the total return on investment project i (V_i) undertaken by firm i depends on two factors. The first one is the specific characteristics of the project measured by x_i and related to the firm. The second one is a macroeconomics parameter $\theta = \overline{\theta} + \sigma_\theta \varepsilon$, measuring the productivity of the project that depends on three elements. A fix parameter $\overline{\theta}$ that is a measure of the average productivity, a macroeconomic shock $\varepsilon \sim N(0,1)$

7. Taking another angle, Jarrow (2013b) shows that the mix of capital adequacy rules based on a risk sensitive model, maximum leverage ratio and stress testing (three approaches proposed in Basel III) may increase the probability of catastrophic financial institution failure.

that is normally distributed, and the sensitivity of the productivity to this shock, σ_θ. Thus, the total return on project i at period 2 is given by $V_i = \theta x_i = \left(\overline{\theta} + \sigma_\theta \varepsilon\right) x_i$. Since by assumption $\mathrm{E}[\varepsilon] = 0$, the expected value of the total return on investment at period 1 is given by $\mathrm{E}[V_i] = \overline{\theta}\, x_i$.

Finally, firms must provide an asset (e.g. land) as collateral for their loan, and $Z^e \in]0,1[$ is the expected value of this collateral for period 2. In the following, we assume that the bank's expectation about the value of the collateral is exogenous and is related to the historical value of the bank's debt recovery rate, but also to the level of the bank's "optimism" (or "pessimism") concerning the future.

There is one bank in the economy endowed with its own capital (the bank is owned by shareholders who provide it with equity capital) and individual investors' deposits. These deposits are insured through a government-funded scheme (full deposit insurance). Consequently, deposits are riskless and the net interest rate on deposits is equal to the risk-free rate, which is set to zero.

According to our previous assumptions, there is no moral hazard between the bank, individual investors and firms since the value of x_i is common knowledge at period 1 and realisation of V_i is freely observable by all parties at period 2.

Let us define R as the rate of return charged by the bank to the projects it finances. At the beginning of period 1, firms apply for credit and, since there is no moral hazard, the bank finances firms as long as the expected value of their projects exceeds the rate of return they must pay back in period 2, such that

$$E[V_i] = \overline{\theta}\, x_i \geq R \text{ with } x_i \in [0,1] \tag{1}$$

We assume that $\overline{\theta} > R$, and from equation (1) it is easy to show that the last firm financed by the bank is given by

$$\overline{x}_i(R) = \frac{R}{\overline{\theta}} < 1 \tag{2}$$

and the total quantity of financing in the economy is given by

$$D(R) = \int_{\overline{x}_i(R)}^{1} 1\, dx_i = \left(1 - \frac{R}{\overline{\theta}}\right) \tag{3}$$

2.2. Bank capital requirements and capital buffer

We assume the bank is subject to capital requirements defined by the Basel Committee on Banking Supervision (2006). Given the international nature of the capital regulation and the national disparities in the definition of bank capital, the Committee separates the components of capital in two parts: the Core Capital and the Supplementary Capital, respectively known as Tier 1 and Tier 2 capital. The Core Capital consists of key elements common to all banking systems that display the highest loss-absorbing capabilities, like bank's equity and disclosed reserves. The Supplementary capital reflects the national disparities in the definition of bank capital components; however the Committee considers the legitimacy of certain elements which are included in the Tier 2 capital. In the model, we assume for simplicity that bank capital only consists of Tier 1, in accordance with the definition retained in order to compute the leverage ratio proposed in Basel III.[8]

Still in accordance with the Basel Committee, we assume that the level of bank's capital requirements is related to its loan portfolio risk class. We retain the Internal Rating-Based approach that exists in two versions: the Foundation IRB and the Advanced IRB approaches. The latter gives a greater degree of autonomy for banks, but they both share the same principle. Under the IRB approaches, banks compute regulatory requirements by applying a regulatory risk weight function that is based on the asymptotic single risk factor model proposed by Vasicek (2002). This risk weight function requires regulatory inputs which are assessed by the banks via their internal data.

Conditional on supervisory accordance, foundation-IRB-banks estimate the Probability of Default (PD_i) of a loan i, and advanced-IRB-banks further assess the Loss Given Default (LGD_i), the Exposure At Default (EAD_i), and the effective Maturity (M_i) for this loan. The synthetic form of the regulatory risk weight function for each loan i is:[9]

$$\text{Bank Regulatory Capital}_i = EAD_i \times LGD_i \times \left(Default\ Rate_i(\alpha_r) - PD_i \right) \times M_i \qquad (4)$$

8. "*The capital measure used for the leverage ratio at any particular point in time is the Tier 1 capital measure applying at that time under the risk-based framework*" (Basel Committee on Banking Supervision, 2016).

9. Basel Committee on Banking Supervision (2006).

The default rate expresses an assessment of the default of a loan for a specific value of the probability of bank's non-default chosen by the regulator (α_r), which is currently equal to 99.9 % $(\alpha_r = 0.999)$. This default rate is defined in Basel III as:[10]

$$Default\ Rate_i(\alpha_r) = \phi\left[\frac{\phi^{-1}(PD_i) + \sqrt{\rho}\,\phi^{-1}(\alpha_r)}{\sqrt{(1-\rho)}}\right] \tag{5}$$

where ϕ denotes the cumulative distribution function of a standard normal random variable and ρ is a correlation parameter, also defined by the regulator.[11] The default rate is increasing with the risk classes of the loan given by its probability of default PD_i, and with the target of non-default probability fixed by regulator (α_r).

We assume that the bank retains the advanced IRB approach proposed by the Basel II/III capital requirements in order to compute its level of regulatory capital.[12] Consequently, four variables must be estimated: the Maturity, the Exposure at Default, the LGD, and the probability of default for each loan (or equivalently each firm) financed by the bank.

According to our assumption, the length of the period (that measures the effective maturity, M) and the size of the loans (that measures the Exposure at Default, EAD) are equal to 1 (see above). Consequently, substituting (5) into (4), the value of bank capital for a loan i is equal to (with $M_i = EAD_i = 1$).

$$\begin{aligned}\text{Bank Regulatory Capital}_i = {} & LGD_i \\ & \times\left(\phi\left[\frac{\phi^{-1}(PD_i) + \sqrt{\rho}\,\phi^{-1}(\alpha_r)}{\sqrt{(1-\rho)}}\right] - PD_i\right)\end{aligned} \tag{6}$$

The LGD is inversely related to the estimated value of the bank's debt recovery rate. Also, this estimated debt recovery rate is positively correlated with the expected value of the collateral provided by firms. Consequently, we assume that the LGD chosen by the bank is a decreasing function of the expected value of the collateral provided by firms with:

10. Basel Committee on Banking Supervision (2006).

11. According to Basel II and III, we have $\rho = 0.12 * \left(\frac{1-e^{-50*PD_i}}{1-e^{-50}}\right) + 0.24 * \left(1 - \frac{1-e^{-50*PD_i}}{1-e^{-50}}\right)$.

12. This is the version retained by most of the large national and international financial institutions.

$$\lambda(Z^e) < 1, \; \frac{\partial\lambda(Z^e)}{\partial Z^e} < 0 \text{ and } \underbrace{\lim}_{Z^e \to 1} \lambda(Z^e) \to 0$$

This assumption means that the higher the expected value of the collateral for period 2 (Z^e), the higher the estimated debt recovery rate and the lower the LGD retained by the bank for computing its level of regulatory capital.

Finally, the last parameter required to compute the level of bank capital requirements is the borrowing firm's probability of default. In our framework, a project *i* is in default (liquidated by the bank) if the firm cannot repay the value *R* at period 2. Formally, the probability of default of each project is given by the following conditional probability:

$$PD_i = P\left[V_i < R \middle| x_i \geq \overline{x}_i(R)\right] \tag{7}$$

Equation (7) is the probability that the final value of project *i* at the end of period 2 is lower than the rate of return charged by the bank, conditional on the fact that the project was financed.

As $V_i = \theta x_i = \overline{\theta} x_i + x_i \varepsilon \sigma_\theta$, equation (7) becomes

$$PD_i = P\left[\overline{\theta} x_i + x_i \varepsilon \sigma_\theta < R \middle| x_i \geq \overline{x}_i(R)\right]$$

Rearranging, we obtain

$$PD_i = P\left[\varepsilon < \overline{\varepsilon}_i(R) = \frac{R - \overline{\theta} x_i}{x_i \sigma_\theta} \middle| x_i \geq \overline{x}_i(R)\right] = \overline{p}_i = \phi(\overline{\varepsilon}_i(R)) \tag{8}$$

where ϕ denotes the cumulative distribution function of a standard normal random variable. Equation (8) means that project *i* (or loan *i*) defaults if the realised value of the shock ε is larger than the critical value $\overline{\varepsilon}_i(R) = \dfrac{R - \overline{\theta} x_i}{x_i \sigma_\theta}$, with $\overline{\varepsilon}_i(R) \leq 0$ since $R \leq \overline{\theta} x_i$. This probability of default is an increasing function of the rate of return (R) charged by the bank, and the sensitivity of the productivity to the shock, σ_θ. Conversely, this probability of default decreases in line with the value of x_i i.e. the intrinsic "quality" of project *i*.

We can substitute $PD_i = \overline{p}_i = \phi(\overline{\varepsilon}_i(R))$ and $\lambda = \lambda(Z^e)$ in equation (6) to obtain the amount of capital required to the bank by the regulator to finance project *i*,[13]

13. Note that $\phi^{-1}(\overline{p}_i) = \phi(\phi(\overline{\varepsilon}_i(R))) = \overline{\varepsilon}_i(R)$.

$$\text{Bank Regulatory Capital}_i = k_{ir} = \lambda(Z^e)\left[\phi\left[\frac{\overline{\varepsilon}_i(R) + \sqrt{\rho}\phi^{-1}(\alpha_r)}{\sqrt{(1-\rho)}}\right] - \phi(\overline{\varepsilon}_i(R))\right]$$

For the sake of simplicity, hereafter we use $P_i(\alpha_r, \overline{\varepsilon}_i(R)) = \phi\left[\frac{\overline{\varepsilon}_i(R) + \sqrt{\rho}\phi^{-1}(\alpha_r)}{\sqrt{(1-\rho)}}\right]$ as the rate of default and bank capital requirements for project *i* are given by the formula

$$k_{ir}(R) = \lambda(Z^e)(P_i(\alpha_r, \overline{\varepsilon}_i(R)) - \phi(\overline{\varepsilon}_i(R))) \tag{9}$$

Finally, the total amount of the bank's capital requirements is equal to the level of capital required in order to cover its loan portfolio in accordance with the Basel regulation:

$$K_r(R) = \int_{\overline{x}_i(R)}^{1} k_{ir}(R)dx_i = \int_{\overline{x}_i(R)}^{1} \lambda(Z^e)(P_i(\alpha_r, \overline{\varepsilon}_i(R)) - \phi(\overline{\varepsilon}_i(R)))\, dx_i \tag{10}$$

Following Heid (2007), we assume that the bank goes bankrupt if its net value at period 2 is lower than the level of capital required by the regulation since in that case, the bank can be shut down by the regulatory authorities.[14] Consequently, in such a framework, the level of capital effectively provided by the bank (or its economic capital) should be higher than the regulatory requirements we have just computed. In fact, as investments returns are random, the bank prefers to operate with a higher level of capital compared to regulatory requirements. The difference between the actual level of bank capital and regulatory requirements is known as the capital buffer. It is important to note that banks first choose their investments portfolio which in turn determines their level of capital and capital buffer.

We define α_b the probability of non-default chosen by the bank and assume that this probability of non-default is higher than the one imposed by the prudential regulation α_r $(\alpha_b > \alpha_r = 0.999)$.[15]

14. Heid (2007, pp. 3888-3889) states that "*regulatory requirements shift the bank's default point from 0 [the solvency constraint] to the regulatory constraint*".

15. In the simulation proposed in part 4, we retain $\alpha_b = 0.9997$ which means that the bank default only in 0.03% a year. This rate of default is in accordance with an AA grade by rating agencies, which is the minimum grade required to operate in the money fund market.

Consequently, substituting α_b in equation (5) we can compute the loan default rate compatible with the probability of non-default chosen by the bank. We obtain

$$Default\ Rate_i(\alpha_b) = \phi\left[\frac{\phi^{-1}(PD_i) + \sqrt{\rho}\phi^{-1}(\alpha_b)}{\sqrt{(1-\rho)}}\right] > Default\ Rate_i(\alpha_r)$$

Finally, using the same value of the parameters we compute for *EAD*, *LGD*, *PD*, *M*, we compute the bank's economic capital and the level of its capital buffer. Substituting the new default rate in equation (10) we obtain

$$K_b(R) = \int_{\bar{x}_i(R)}^{1} k_{ib}(R)dx_i = \int_{\bar{x}_i(R)}^{1} \lambda(Z^e)(P_i(\alpha_b, \bar{\varepsilon}_i(R)) - \phi(\bar{\varepsilon}_i(R)))dx_i \quad (11)$$

The value of bank's capital or economic capital $(K_b(R))$ is always higher than the value of regulatory capital given by $K_r(R)$ and the capital buffer is equal to

$$\hat{b} = K_b(R) - K_r(R) > 0 \quad (12)$$

In other words, K_r (the regulatory capital) is similar to the level of capital that ensures bank solvability and it is always binding since it is lower than the effective level of bank capital K_b.

3. Equilibrium and financial fragility

In equilibrium, the bank chooses the quantity of projects financed (its loans portfolio) and determines the amount of capital required to cover the risk of this portfolio according to the prudential regulation and to its own non-default probability (α_b) target. As we assume that the bank is more conservative than what is required by the regulation, the level of bank capital is higher than the level of regulatory capital and capital buffer is positive. Finally, the amount of deposit is determined by the difference between the value of the bank's loans portfolio and the amount of bank economic capital, which also determines its level of leverage. In such a framework, all equilibrium quantities will be expressed as a function of the rate of return charged by the bank.

3.1. Bank's equilibrium and the total value of financing

As in Blum (1999), we assume that the bank perfectly act in the interest of shareholders. As a consequence, the bank bases its capital allocation process on its shareholder value and maximises its net expected discounted value.

The bank's expected profit depends on the number of projects it finances and the number of defaulting loans. When the bank finances a project, it expects to receive the rate of return R if the project succeeds at period 2 and the expected value of the collateral for period 2, (Z^e) if the project fails.[16] Project *i*'s probability of default is given by $\phi(\overline{\varepsilon}_i(R))$ and its probability of success is given by $(1-\phi(\overline{\varepsilon}_i(R)))$. The last project financed by the bank is given by $\overline{x}_i(R)$ and the bank's deposit cost is assumed to be zero.

Thus, the expected profit of the bank at period 1, net of the cost of the funds, is equal to

$$\Pi_b^e(R) = \int_{\overline{x}_i(R)}^{1} [(1-\phi(\overline{\varepsilon}_i(R)))R + \phi(\overline{\varepsilon}_i(R))Z^e]dx_i$$

$$- \int_{\overline{x}_i(R)}^{1} [(1-k_{ib}(R))]dx_i \text{ or}$$

$$\Pi_b^e(R) = \int_{\overline{x}_i(R)}^{1} [(1-\phi(\overline{\varepsilon}_i(R)))R + \phi(\overline{\varepsilon}_i(R))Z^e - 1]dx_i + \int_{\overline{x}_i(R)}^{1} k_{ib}(R)dx_i$$

As $K_b(R) = \int_{\overline{x}_i(R)}^{1} k_{ib}(R)dx_i$ we have

$$\Pi_b^e(R) = \int_{\overline{x}_i(R)}^{1} [(1-\phi(\overline{\varepsilon}_i(R)))R + \phi(\overline{\varepsilon}_i(R))Z^e - 1]dx_i + K_b(R) \quad (13)$$

The bank's objective is to maximize its net expected present value defined as the discounted value of the net expected profit. Define $(1+\delta) > 1$ as the discounted rate of profit, we have:

16. We assume that a project has no residual value in the event of default.

$$V_b^e(R) = \frac{1}{(1+\delta)}\left[\int_{\bar{x}_i(R)}^{1} [(1-\phi(\bar{\varepsilon}_i(R)))R + \phi(\bar{\varepsilon}_i(R))Z^e - 1]dx_i + K_b(R)\right] \quad (14)$$

From equation (14), we can see immediately that the rate of return charged by the bank (R) has an ambiguous effect on its net expected present value. On the one side, a rise in R will increase the profitability of the bank since it leads to a higher return from each successful project. On the other side, since firms' default probability $(\phi(\bar{\varepsilon}_i(R)))$ is an increasing function of the rate of return, a rise in R will decrease the profitability of the bank because it will lead to a higher probability of default for each project financed. Note also that the higher the firms' probability of default, the larger the quantity of capital provided by the bank. The balance between these two opposite forces – rise in profitability, and rise in default and capital – determines the equilibrium value of the rate of return that maximises the bank net expected present value.

Proposition 1 gives the formal condition for the existence of the rate of return that maximises equation (14). It also indicates the equilibrium level of leverage compatible with net expected present value maximisation.

Proposition 1.

a. For $\bar{\theta} > 2 - Z^e$ and σ_θ sufficiently small, there is a unique value $R^* \in \left]R_c, \bar{\theta}\right[$ with $R_c > 1$ that maximises the bank's net expected present value and $V_b^e(R^*) > 0$.

b. The total level of financing in the economy is given by $D^*(R^*) = \int_{\bar{x}_i(R^*)}^{1} 1dx_i = \left(1 - \frac{R^*}{\bar{\theta}}\right)$ with $\frac{\partial D(R)}{\partial R} < 0, \forall R$ and the equilibrium level of the bank's leverage (or equity multiplier) is equal to $\ell^*(R^*) = \frac{D^*(R^*)}{K_b^*(R^*)}$ with $\frac{\partial \ell(R)}{\partial R} < 0, \forall R$.

Proof of Proposition 1: see Appendix 1.

According to part (a) of proposition 1, the bank's net expected present value is maximised for a unique value of the rate of return it charges to firms. This value depends on the expected value of the collateral for period 2 (Z^e), and the value of $\overline{\theta}$ and σ_θ which can be understood as proxying for the overall business climate.

In addition, as stated in part (b) of proposition 1, the total financing in the economy is a decreasing function of the rate of return charged by the bank, since when the bank's rate of return on loans falls, new firms will be financed. Finally, bank leverage is a decreasing function of the rate of return it charges to firms. This result is straightforward since firms are financed partly by bank capital and partly by deposits. Thus, when the bank cuts the rate of return it charges to firms, its level of assets increases (since the amount of loans financed increases) at a faster pace than its level of regulatory capital, leading to a rise in its equilibrium level of leverage.

3.2. Bank leverage and financial fragility

In this section, we identify the relationship between the level of bank leverage and bank financial fragility. We refer to bank's financial fragility as the bank's likelihood of bankruptcy and we follow Heid (2007) by assuming that the bank will default if its value at period 2 is lower than the level of capital required by the regulation.

The value of the bank at period 2 has two components. The first part is the capital endowment allowing the bank to absorb part of the firms' default resulting from the macroeconomic shock. The second part is determined by the value of the bank's assets.

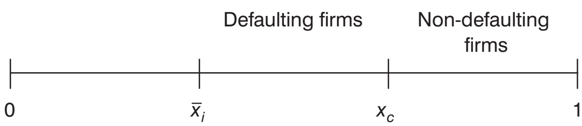
Bank capital can be splitted into regulatory capital and capital buffer. Both are fixed at period 1 for period 2 according to the level of risk of the bank loans portfolio and its level of non-default probability. The value of the bank's assets is linked to the realised value of the macroeconomic shock and the effective value of the collateral at period 2, which we label $\overline{Z}$ as opposed to Z^e which is the expected value of the collateral at period 1 for period 2.

We make two assumptions at this stage. First, the effective value and the expected value of the collateral may differ. In fact, if the bank is pessimistic or optimistic regarding the future, the expected value of the collateral will be respectively lower or higher than the effective value. Second, for the sake of simplicity, we assume that the effective value of the collateral is independent of realisation of the macroeconomic shock.

Under these assumptions, the bank's value at period 2 depends on the macroeconomic shock.

Let us define $\varepsilon_c < 0$ as the value of the macroeconomic shock at which firms i with $\bar{x}_i < x_i \leq x_c = \dfrac{R}{\bar{\theta} + \sigma_\theta \varepsilon_c}$ will default. This means that financed firms with $x_i \in \left]x_c, 1\right]$ are successful, while financed firms with $x_i \in \left[\bar{x}_i, x_c\right]$ are in default (see figure 1 for a graphical illustration).

Figure 1. Allocation between defaulting and non-defaulting loans



The value of the bank at period 2 is thus given by

$$V_b(R) = \int_{x_c(R)}^{1} R dx_i + \int_{\bar{x}_i(R)}^{x_c(R)} \bar{Z} dx_i - \left[\int_{\bar{x}_i(R)}^{1} 1 dx_i - K_b(R) \right] \tag{15}$$

Equation (15) means that the bank earns R for each non-defaulting loan, $\bar{Z}$ for each defaulting loan, and must reimburse deposits for the whole value of its loans portfolio. According to our assumption, the bank goes into bankruptcy when its effective value at period 2 is lower than the level of capital required by the regulation:

$$V_b(R) < K_r(R) \quad \text{or} \quad V_b(R) - K_r(R) < 0$$

As $\hat{b} = K_b(R) - K_r(R)$, we have

$$V_b(R) - K_r(R) = \int_{x_c(R)}^{1} R dx_i + \int_{\bar{x}_i(R)}^{x_c(R)} \bar{Z} dx_i - \int_{\bar{x}_i(R)}^{1} 1 dx_i + \hat{b} \ \text{ or}$$

$$V_b(R) - K_r(R) = R[1 - x_c(R)] + \bar{Z}[x_c(R) - \bar{x}_i(R)] - [1 - \bar{x}_i(R)] + \hat{b}$$

Consequently, the bank goes into bankruptcy when

$$R[1 - x_c(R)] + \bar{Z}[x_c(R) - \bar{x}_i(R)] - [1 - \bar{x}_i(R)] + \hat{b} < 0$$

Or equivalently with $x_c(R) = \dfrac{R}{\overline{\theta} + \varepsilon_c \sigma_\theta}$ and $\overline{x}_i(R) = \dfrac{R}{\overline{\theta}}$

$$R\left[1 - \frac{R}{\overline{\theta} + \varepsilon_c \sigma_\theta}\right] + \overline{Z}\left[\frac{R}{\overline{\theta} + \varepsilon_c \sigma_\theta} - \frac{R}{\overline{\theta}}\right] - \left[1 - \frac{R}{\overline{\theta}}\right] + \hat{b} < 0 \quad (16)$$

Equation (16) depends on the effective value of the collateral, the rate of return chosen by the bank, the value of the capital buffer and the state of the business climate $(\overline{\theta}, \sigma_\theta)$. For a given value of these variables, it is possible to determine the value of the macroeconomic shock at which the bank goes into bankruptcy. This main result is given in proposition 2.

Proposition 2.

a. When $\varepsilon < \varepsilon_c$ with $\varepsilon_c \equiv \dfrac{\overline{\theta}\left[(R-1)\left(R-\overline{\theta}\right) - \hat{b}\overline{\theta}\right]}{\sigma_\theta\left[R\left(1-\overline{Z}\right) + \overline{\theta}(R-1) + \hat{b}\overline{\theta}\right]}$ the bank goes into bankruptcy.

b. There is a value $R_{\min}$ of the rate of return associated with a value of bank leverage $\ell(R_{\min})$ that minimises the bank's probability of default.

Proof of Proposition 2: see Appendix 2.

The intuition for proposition 2 is straightforward. The value ε_c can be considered as a measure of the financial fragility of the economy because it defines the critical level of the macroeconomic shock for which the bank is bankrupted. A rise in ε_c means that the bank is more sensitive to a shock in the sense that the value of the shock that is required to make it fail is lower: financial fragility increases. In our model, the degree of financial fragility depends on the overall business climate $(\overline{\theta}, \sigma_\theta)$, the effective value of the collateral $(\overline{Z})$, the rate of return charged by the bank (R) and the level of bank capital buffer $(\hat{b})$. It is straightforward that a rise in the level of capital buffer increases financial soundness of the bank and leads to a drop in the value of ε_c.

In fact, the balance between two coexisting forces determines a nonlinear relationship between the rate of return charged by the bank and financial fragility.

In order to understand this result, assume first that the rate of return charged by the bank is high. According to proposition 1, the value of leverage and the total quantity of financing are low. Assume now that the bank decides to cut the value of the rate of return it charges to firms. Two mechanisms that work in opposite directions come into play. First, firms' *ex-ante* probability of default falls with the rate of return charged by the bank. This mechanism plays positively on bank soundness as it reduces bank financial fragility. Second, as the rate of return charged by the bank decreases, more firms are financed and the bank's level of capital increases with the quantity of financing. However, since firms are financed partly by bank capital and partly by deposits, the value of leverage increases (see part b. of proposition 1). As leverage increases, the *ex-post* value of the bank becomes more dependent on the value of its assets. This second mechanism plays negatively on bank soundness and increases bank financial fragility since the *ex-post* value of the bank's assets is related to the level of the macroeconomic shock. Consequently, there is a critical value of the rate of return $(R_{\min})$ charged by the bank beyond which the second effect outweighs the first effect, and the bank becomes more sensitive to the value of the macroeconomic shock. This critical value of the rate of return charged by the bank is associated with a critical value of leverage $\ell(R_{\min})$ at which financial fragility increases in line with leverage.

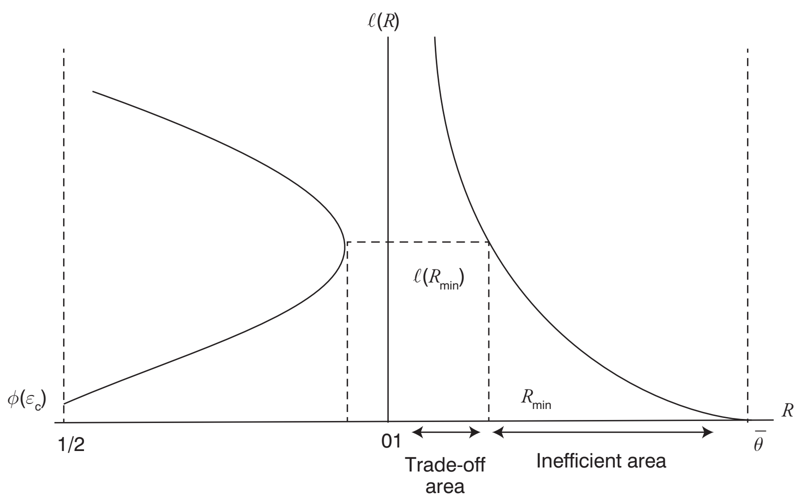
$\ell(R_{\min})$ is thus defined as the "minimum fragility leverage value" which means the value of the equity multiplier at which the bank's probability of default is at its minimum. However, there is no reason for the bank to choose this specific value. On the contrary, we have shown that the equilibrium level of leverage chosen by the bank is the level that maximises its net expected present value, $\ell(R^*)$. Consequently, two situations are possible. In the first case, $\ell(R^*) < \ell(R_{\min})$ and the equilibrium value of leverage chosen by the bank is lower than the "minimum fragility leverage value". This situation is inefficient from the point of view of the economy as a whole, since it is possible to increase the quantity of financing and to reduce financial fragility. A lower rate of return charged by the bank will increase the amount of funds available to firms. Simultaneously, this increase in the quantity of financing will lead to an increase in the level of bank leverage and a decrease in the bank's probability of default. In the second case, $\ell(R^*) > \ell(R_{\min})$ and the equilibrium value of leverage is higher than the "minimum fragility leverage value". In this case, there is a trade-off between financial fragility and credit availability since a higher degree of financial fragility must be accepted in order to increase the quantity of credit available to firms above $\ell(R_{\min})$.[17]

17. This result is in line with Inderst and Mueller (2008) who show that leverage is beneficial, at least up to a certain point, to provide an incentive for banks to make new risky loans.

Figure 2 provides a graphical representation of this mechanism. The right quadrant describes the relationship between the rate of return charged by the bank and the value of bank leverage $\ell(R)$. This relationship is decreasing since a decline in the rate of return charged by the bank leads to a rise in leverage (proposition 1). The left quadrant links the value of bank leverage to its probability of default $(\phi(\varepsilon_c))$, which is related to the critical value of the macroeconomic shock ε_c (proposition 2).

As the rate of return charged by the bank decreases, the amount of funds available to firms increases and more projects can be undertaken. Simultaneously, this increase in the quantity of financing leads to a rise in the level of bank leverage and a decrease in the probability of default as long as $\ell(R) < \ell(R_{\min})$. When the level of bank leverage becomes higher than $\ell(R_{\min})$, the bank's probability of default increases in line with the level of financing. Consequently, from that point, higher credit availability is possible if one accepts a higher level of financial fragility.

Figure 2. Leverage and bank financial fragility



4. Rise in expected value of collateral, financial fragility and macroprudential regulation

So far, we have shown that the equilibrium rate of return charged by the bank depends on the expected value of the collateral for period 2, and the overall business climate of the economy. Below, we study the impact of a change in the expected value of the collateral on bank behaviour and financial fragility.

First, we show that the bank's equilibrium level of leverage increases with the expected value of the collateral. Second, we show there is a critical threshold for the expected value of the collateral after which the equilibrium value of bank leverage becomes higher than the "minimum fragility leverage value". This result implies that, above this threshold, bank financial fragility increases with the expected value of the collateral. Therefore, we estimate "the minimum fragility leverage value" for given parameter values. This heuristic experiment shows that the level of leverage that "minimizes fragility" might be far from the maximum leverage value fixed by the regulator and can vary with the overall business climate.

4.1. Rise in collateral's expected value and financial fragility

It is possible to show that a rise in the expected value of the collateral has a positive impact on the total level of financing and the equilibrium value of leverage chosen by the bank. We show also that, after a given threshold, an increase in the expected value of the collateral and bank leverage rises financial fragility.

Proposition 3

a. $\ell(R^*)$ is an increasing function of the expected value of the collateral Z^e.

b. There is a critical value Z_c^e for which $\ell(R^*) > \ell(R_{\min})$ and financial fragility increases along with the rise in the expected value of the collateral.

Proof of proposition 3: see Appendix 3.

The two parts of proposition 3 are straightforward.

First, a rise in the expected value of the collateral has a direct positive impact on the bank's net expected profit since, *ceteris paribus,* it increases the expected return in the event of firm default. Note also that a change in the expected value of the collateral directly alters the required level of regulatory capital (it decreases), since it depends on the Loss Given Default (LGD) value estimated by the bank in the advanced IRB model. Consequently, there is a kind of "freeing" of the amount of economic capital compared to the previous situation, and the bank has to change its behaviour in order to reach a new equilibrium. The "freeing" in the level of capital, and the increase in the net expected profit for each loan that is financed, means that the bank is inclined to increase its level of financing. This can be done by cutting the rate of return charged on each loan. In this case, the amount of funds provided to firms increases and the *ex-ante* probability of default of each project falls as the rate of return charged to each firm decreases. At the same time, as the quantity of financing increases, the required level of bank's capital increases. This process stops as soon as the bank has restored the equilibrium value of its net expected present value. Lastly, the equilibrium level of leverage increases in line with the quantity of financing (see proposition 1).

Second, as the equilibrium value of leverage chosen by the bank increases with the rise in the expected value of the collateral, there is a critical expected value of the collateral at which the bank's effective leverage becomes higher than the "minimum fragility leverage value" (part b. of proposition 3). This result is straightforward since the "minimum fragility leverage value" depends on the effective value of the collateral, which is different (and generally lower) than the expected one. This means that financial fragility increases with the rise in the expected value of the collateral because the bank becomes increasingly sensitive to macroeconomic shocks.

However, because of the structure of the model, it is impossible to compute this critical expected value of collateral. Thus, in the last part of the paper, we provide a numerical illustration of proposition 3. This illustration is purely heuristic in the sense that we do not consider it a prescriptive tool, but rather a way of stressing that both the "minimum fragility leverage value" and the critical expected value of the collateral above which financial fragility increases, depend on the overall business climate and may differ from the value proposed by the regulator.

4.2. A numerical illustration

We have underlined that the Basel Committee has chosen a minimum leverage ratio of 3%, and thus a maximum equity multiplier of 33. These values seem to be consistent with the historical averages in non-crisis periods, but they are not based on specific economic reasoning (Jarrow, 2013a). In this part of the paper, we provide a numerical illustration of proposition 3 in order to show that, for some plausible values of the various parameters of the model, the "minimum fragility leverage value" is far from 33.

Table 1 presents the parameters values adopted for the simulation. As our main objective is to illustrate the possible rises of financial fragility of the banking system in situation of confidence (what we name a "good business climate"), we retain values of parameters in line with this assumption. In appendix 4, we propose another simulation with values of parameters compatible with "bad business climate".

Table 1. Values of the parameters for a "good business climate"

$\overline{\theta}$	σ_θ	$\bar{Z}$	$(1+\delta)$
1.55	0.05	0.55	1.05

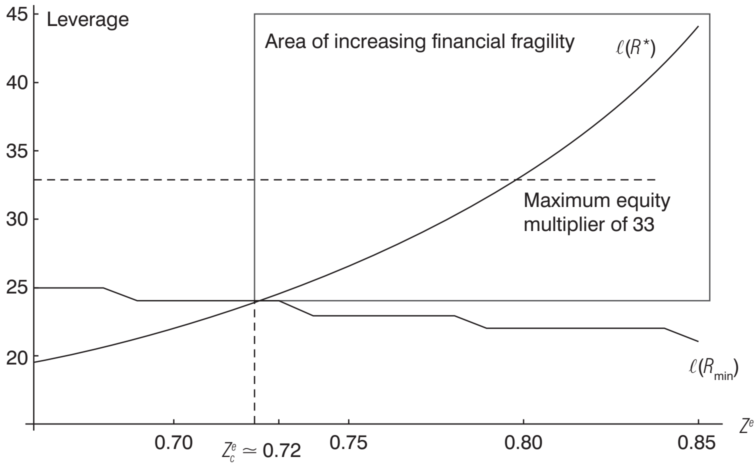
The discounted rate of profit is set to 5%. We retain a debt recovery rate $\left(\bar{Z}\right)$ of 55% which is compatible with the average mean recovery rate observed in Moody's or S&P's reports during financial crises.[18] The sensitivity of the productivity to the macroeconomic shock (σ_θ) and the maximum rate of return on financed projects $\left(\overline{\theta}\right)$ are chosen to be compatible with a "good business climate" (5.0% and 55% respectively).

Figure 3 is a graphical representation of the "minimum fragility leverage value" and effective levels of leverage retained by the bank according to the expected value of the collateral. It provides a numerical illustration of proposition 3 for the values shown in Table 1. The "minimum fragility leverage value" $(\ell(R_{\min}))$ corresponds to the decreasing line in figure 3. For the retained values, it is well below the maximum equity multiplier of 33 fixed by Basel III prudential regulation (dotted line in figure 3). The increasing function represents the various equilibrium values of leverage chosen by the bank $(\ell(R^*))$ according to the expected value of the collateral, for the range $Z^e \in [0.66;0.85]$.[19] It is possible to define graphically the "area of increasing financial fragility" as the equilibrium situations where the level of leverage chosen by the bank (the level that maximises its profit) is higher than the "minimum fragility leverage value". This analysis implies that, outside this area, a rise in bank's leverage reduces financial fragility whereas within this area, a rise in bank's leverage results in a rise in financial fragility as the likelihood of bank bankruptcy increases. This "area of increasing financial fragility" is bounded by the critical expected value of the collateral which, in this case, is about 0.72 (for a level of leverage around 23.94).

18. Moodys Annual Default Study, 2012.

19. Simulations are performed using Mathematica. The program is available from the authors on request.

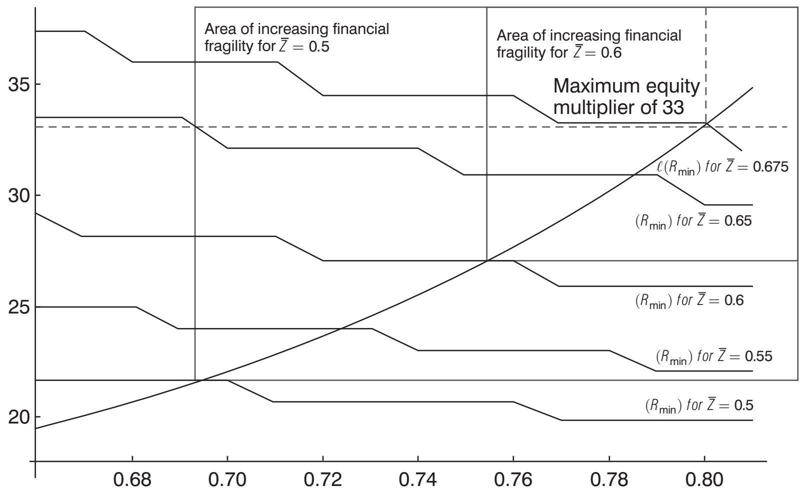
Figure 3. Level of leverage and area of increasing financial fragility for "good business climate"



This result highlights that the choice of a fixed regulatory level for the leverage ratio could be misleading if the objective is to reduce bank's financial fragility. Under specific macroeconomic conditions or a specific business climate, a bank may choose a level of leverage lower than that fixed by the new regulation, but higher than the "minimum fragility leverage value" compatible with the economic situation. It is also clear that the "minimum fragility leverage value" $(\ell(R_{\min}))$ is an increasing function of the effective debt recovery rate $\left(\bar{Z}\right)$, as shown in figure 4.

It means that the size of the "area of increasing financial fragility" decreases with the value of the effective recovery rate. Consequently, it is possible to determine the value of the effective recovery rate for which this area appears only for a leverage higher than the maximum equity multiplier (area in dotted line in figure 4). This happens, in figure 4, for $\bar{Z} \simeq 0.675$. In such a case, the regulatory constraint is binding (as bank's leverage cannot be higher than the maximum equity) and the bank will necessary choose, at the equilibrium point, a level of leverage outside the area of increasing financial fragility.

Figure 4. Minimum fragility leverage value according effective debt recovery rate



5. Conclusion

We have shown in this paper that financial fragility can emerge even if it is assumed that banks make rational decisions under perfect information. Our analysis provides two main results. First, we show that risk-sensitive microprudential regulation, such as Basel II or III, cannot prevent an increase in financial fragility associated with bank behaviour. In periods of economic expansion, characterised by a rise in assets prices, optimal bank behaviour leads to an increase in leverage that heightens financial fragility. Consequently, a maximum leverage ratio constraint seems justified in order to prevent financial fragility. The new Basel III macroprudential regulations address this issue and aims to impose maximum bank leverage (equity ratio) of 33. However, our second result highlights that the value of leverage that minimizes financial fragility is not constant along the cycle. This results implies the regulator should adjust the leverage ratio in order to be effective. This result is in line with Basel III regulatory innovations including countercyclical capital ratios leading to stricter capital requirements during boom periods in order to restrict the supply of loans (Tavman, 2015). In Basel III, these countercyclical provisions must be calibrated not according to the specific exposure of each financial institution, but in response to the total exposure relating to the stage in the economic cycle. Rather than resorting to totally discretionary devices as might apply to an enriched Pillar 2 within Basel III or, conversely, adopting automatic rules, the option chosen by the Basel Committee is to define guidelines or targets (for instance the value of total loans on GDP), which if exceeded might justify a gradual increase in the capital requirements of Pillar 1. Since our model shows that the value of leverage that minimizes financial fragility is not constant within the cycle, we would advocate for a similar approach based on targets to define an adjustable leverage ratio. Our results support targets based on the level of asset prices and macroeconomic volatility, in accordance with Calmès and Théoret (2013). They conduct an empirical study of the efficiency of a leverage ratio for Canadian banks. They find that "*Although the Basel III leverage ratio can help monitor bank leverage, (…) additional measures should be considered to get a clearer picture of bank leverage, each providing complementary information on the stance of bank risk at various time frequencies.*" (p. 30). The analysis in this paper could be extended in several directions. First, our analysis deals with only one aspect of systemic risk created by banks' behaviour, namely *aggregate systemic risk*, whereas there are two main sources of financial instability linked to two kinds of systemic risk (Bank of England, 2009). The first is the tendency for the bank to take excessive risks in periods of economic expansion. This mechanism leads to the emergence of *aggregate systemic risk* due to the collective tendency of banks to take excessive risk and adopt high leverage levels during expansionary periods in the cycle. The second is the underestimation of spillovers in the banking system leading to *network systemic risk*. This systemic risk is the outcome of common exposure and interconnections among financial institutions, and is due to the sharp increase in funding markets and interbank flows, especially since 2002. In this paper, we addressed only the first source of bank distress, which led to our proposal to establish the leverage ratio. However, we do not underestimate the danger of network systemic risk and consequent contagion mechanisms for triggering a systemic crisis. We believe that this contagion process could amplify the main risk taking mechanisms proposed in this paper. However, because of our specific, one bank framework, these networks effects cannot be properly modeled. Second, we ignore the literature on systemic risk and financial fragility based on securitization and untruthful declarations by banks (Blum, 2008). As we stress in the introduction, our focus is on the possible increase in financial fragility in a perfect framework with no “perturbation” due to informational problems between banks and the regulator. However, untruthful declarations or securitization could amplify financial instability by adding to the main cause of the financial instability (higher leverage) proposed in this paper (Delis and Staikouras, 2011).[20]

20. They show that effective supervision and market discipline requirements are important and complementary mechanisms in reducing bank fragility.

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Appendix

Appendix 1. Proof of proposition 1.

Preliminary

Recall that the level of bank capital is given by

$$K_b(R) = \int_{\bar{x}_i(R)}^{1} k_{ib}(R)dx_i = \int_{\bar{x}_i(R)}^{1} \lambda(Z^e)(P_i(\alpha_b, \bar{\varepsilon}_i(R)) - \phi(\bar{\varepsilon}_i(R)))dx_i$$

(equation (10))

We define $B(R)$ as the net expected profit of the bank

$$B(R) = \int_{\bar{x}_i(R)}^{1} \left[(1 - \phi(\bar{\varepsilon}_i(R)))R + \phi(\bar{\varepsilon}_i(R))Z^e - 1\right]dx_i$$

With $\bar{x}_i(R) = \frac{R}{\bar{\theta}}$, $\bar{\varepsilon}_i(R) = \frac{R - \bar{\theta}x_i}{x_i \sigma_\theta}$

and $P_i(\alpha_b, \bar{\varepsilon}_i(R)) = \phi\left[\frac{\bar{\varepsilon}_i(R) + \sqrt{\rho}\,\phi^{-1}(\alpha_b)}{\sqrt{(1-\rho)}}\right]$ defined previously.

Note that $K_b(R)$ and $B(R)$ are continuous and differentiable on $R \in \left[1, \bar{\theta}\right]$.

We know that for $R \to F(R) = \int_{w(R)}^{u(R)} f(R,x)dx$ we have

$$\begin{aligned}\frac{\partial F(R)}{\partial R} &= F'_R(R) \\ &= \int_{w(R)}^{u(R)} \frac{\partial f}{\partial R}(R,x)dx + u'_R(R) \cdot f[R, u(R)] - w'_R(R) \cdot f[R, w(R)]\end{aligned}$$

Consequently, the partial derivatives of $K_b(R)$ and $B(R)$ relative to R are given by the following two equations.

$$\frac{\partial K_b(R)}{\partial R} = K'_{bR}(R)$$

$$= \int_{\frac{R}{\overline{\theta}}}^{1} \lambda(Z^e)\left(\left(\frac{\partial P_i(\alpha_b,\overline{\varepsilon}_i(R))}{\partial \overline{\varepsilon}_i(R)} - \frac{\partial \phi(\overline{\varepsilon}_i(R))}{\partial \overline{\varepsilon}_i(R)}\right)\frac{\partial \overline{\varepsilon}_i(R)}{\partial R}\right)dx_i$$

$$- \lambda(Z^e)\frac{1}{\overline{\theta}}[P_i(\alpha_b,0) - \phi(0)]$$

$$K'_{bR}(R) = \lambda(Z^e)\left[\int_{\frac{R}{\overline{\theta}}}^{1}\left[\left(P_i(\alpha_b,\overline{\varepsilon}_i(R))'_{\varepsilon} - \phi'_{\varepsilon}(\overline{\varepsilon}_i(R))\right)\frac{1}{x_i\sigma_\theta}\right]dx_i\right.$$

$$\left. - \frac{1}{\overline{\theta}}[P_i(\alpha_b,0) - \phi(0)]\right]$$

$$\frac{\partial B(R)}{\partial R} = B'_R(R) = \int_{\frac{R}{\overline{\theta}}}^{1}\left[(1 - \phi(\overline{\varepsilon}_i(R))) - R\frac{\partial \phi(\overline{\varepsilon}_i(R))}{\partial \overline{\varepsilon}_i(R)}\frac{\partial \overline{\varepsilon}_i(R)}{\partial R}\right.$$

$$\left. + Z^e\frac{\partial \phi(\overline{\varepsilon}_i(R))}{\partial \overline{\varepsilon}_i(R)}\frac{\partial \overline{\varepsilon}_i(R)}{\partial R}\right]dx_i - \frac{1}{\overline{\theta}}[(1 - \phi(0))R + \phi(0)Z^e - 1]$$

$$B'_R(R) = \int_{\frac{R}{\overline{\theta}}}^{1}\left[(1 - \phi(\overline{\varepsilon}_i(R))) - (R - Z^e)\frac{\phi'_{\varepsilon}(\overline{\varepsilon}_i(R))}{x_i\sigma_\theta}\right]dx_i$$

$$- \frac{1}{\overline{\theta}}\left[\frac{1}{2}(R + Z^e) - 1\right]$$

With $\phi(0) = \frac{1}{2}$, $\overline{x}'_i(R) = \frac{\partial \overline{x}_i(R)}{\partial R} = \frac{1}{\overline{\theta}} > 0$ since $\overline{x}_i(R) = \frac{R}{\overline{\theta}}$, $\frac{\partial \phi(\overline{\varepsilon}_i(R))}{\partial \overline{\varepsilon}_i(R)} = \phi'_{\varepsilon}(\overline{\varepsilon}_i(R)) > 0$, $\frac{\partial \overline{\varepsilon}_i(R)}{\partial R} = \frac{1}{x_i\sigma_\theta}$ as $\overline{\varepsilon}_i(R) = \frac{R - \overline{\theta}x_i}{x_i\sigma_\theta}$ and $\frac{\partial P_i(\alpha_b,\overline{\varepsilon}_i(R))}{\partial \overline{\varepsilon}_i(R)} = P_i(\alpha_b,\overline{\varepsilon}_i(R))'_{\varepsilon} > 0$.

Lemma 1.

For $2 - Z^e < \overline{\theta}$, $B(R)$ is strictly concave and there is $R_c \in \left[1, \overline{\theta}\right]$ such that:

$$B(R_c) = 0\,,\ B'_R(R_c) > 0 \text{ and } B(R) \geq 0, \forall R \in \left[R_c, \overline{\theta}\right].$$

Proof of Lemma 1.

We know that $B(R) = \int\limits_{\overline{x}_i(R)}^{1} \left[(1 - \phi(\overline{\varepsilon}_i(R)))R + \phi(\overline{\varepsilon}_i(R))Z^e - 1\right] dx_i$

For $R = 1$ we have $B(1) = \int\limits_{\frac{1}{\overline{\theta}}}^{1} \left[(1 - \phi(\overline{\varepsilon}_i(1))) + \phi(\overline{\varepsilon}_i(1))Z^e - 1\right] dx_i$

< 0 since $Z^e < 1$.

For $R = \overline{\theta}$ we have $B\left(\overline{\theta}\right) = \int\limits_{1}^{1} \left[\left(1 - \phi\left(\overline{\varepsilon}_i\left(\overline{\theta}\right)\right)\right)\overline{\theta} + \phi\left(\overline{\varepsilon}_i\left(\overline{\theta}\right)\right)Z^e\right.$

$\left. - 1\right] dx_i = 0$.

Moreover,

$$B'_R(R) = \int\limits_{\frac{R}{\overline{\theta}}}^{1} \left[(1 - \phi(\overline{\varepsilon}_i(R))) - (R - Z^e)\frac{\phi'_\varepsilon(\overline{\varepsilon}_i(R))}{x_i \sigma_\theta}\right] dx_i$$
$$- \frac{1}{\overline{\theta}}\left[\frac{1}{2}(R + Z^e) - 1\right]$$

and for $R = \overline{\theta}$ we have

$$B'_R\left(\overline{\theta}\right) = \int\limits_{1}^{1} \left[\left(1 - \phi\left(\overline{\varepsilon}_i\left(\overline{\theta}\right)\right)\right) - \left(\overline{\theta} - Z^e\right)\frac{\phi'_\varepsilon\left(\overline{\varepsilon}_i\left(\overline{\theta}\right)\right)}{x_i \sigma_\theta}\right] dx_i$$
$$- \frac{1}{\overline{\theta}}\left[\frac{1}{2}\left(\overline{\theta} + Z^e\right) - 1\right]$$

We know that $\int\limits_{1}^{1} \left[\left(1 - \phi\left(\overline{\varepsilon}_i\left(\overline{\theta}\right)\right)\right) - \left(\overline{\theta} - Z^e\right)\frac{\phi'_\varepsilon\left(\overline{\varepsilon}_i\left(\overline{\theta}\right)\right)}{x_i \sigma_\theta}\right] dx_i = 0$, and for $\overline{\theta} > 2 - Z^e$ we have $\frac{1}{2}\left(\overline{\theta} + Z^e\right) - 1 > 0$.

Consequently, we can conclude that $B'_R(\overline{\theta}) = -\frac{1}{\overline{\theta}}\left[\frac{1}{2}(\overline{\theta} + Z^e) - 1\right] < 0$ for $\overline{\theta} > 2 - Z^e$.

Finally,

$$B''(R) = -\int_{\frac{R}{\overline{\theta}}}^{1} \frac{\phi'_\varepsilon(\overline{\varepsilon}_i(R))}{x_i\sigma_\theta}\,dx_i - \frac{1}{2\overline{\theta}} - \int_{\frac{R}{\overline{\theta}}}^{1}\left[\frac{\phi'_\varepsilon(\overline{\varepsilon}_i(R))}{x_i\sigma_\theta} + (R - Z)\frac{\phi''_\varepsilon(\overline{\varepsilon}_i(R))}{(x_i\sigma_\theta)^2}\right]dx_i - \frac{1}{2\overline{\theta}}$$

which leads, after simplification, to

$$B''(R) = -2\int_{\frac{R}{\overline{\theta}}}^{1} \frac{\phi'_\varepsilon(\overline{\varepsilon}_i(R))}{x_i\sigma_\theta}\,dx_i - \frac{1}{\overline{\theta}} - \int_{\frac{R}{\overline{\theta}}}^{1}(R - Z^e)\frac{\phi''_\varepsilon(\overline{\varepsilon}_i(R))}{(x_i\sigma_\theta)^2}\,dx_i < 0, \forall R \in [1,\overline{\theta}]$$

since $\frac{\phi'_\varepsilon(\overline{\varepsilon}_i(R))}{x_i\sigma_\theta} > 0; \phi''_\varepsilon(\overline{\varepsilon}_i(R)) > 0; \frac{1}{\overline{\theta}} > 0; (R - Z^e) > 0$, for $\overline{\varepsilon}_i(R) \le 0$ which is always the case.

As $B(1) < 0$, $B(\overline{\theta}) = 0$, $B'_R(\overline{\theta}) < 0$ and $B''(R) < 0$, we can conclude that $B(R)$ is strictly concave and there is $R_c \in [1,\overline{\theta}]$ such that $B(R_c) = 0$, $B'_R(R_c) > 0$ and $B(R) > 0$ for $R \in [R_c,\overline{\theta}]$.

The proof of Lemma 1 is completed.

Lemma 2.

For σ_θ sufficiently small we have $K'_{bR}(R_c) > 0$ and $K'_{bR}(\overline{\theta}) < 0$.

Proof of Lemma 2.

We have

$$K'_{bR}(R) = \lambda(Z^e)\left[\int_{\frac{R}{\overline{\theta}}}^{1}\left[\left(P_i(\alpha_b,\overline{\varepsilon}_i(R))'_\varepsilon - \phi'_\varepsilon(\overline{\varepsilon}_i(R))\right)\frac{1}{x_i\sigma_\theta}\right]dx_i - \frac{1}{\overline{\theta}}[P_i(\alpha_b,0) - \phi(0)]\right]$$

For $R = \overline{\theta}$ we have:

$$K'_{bR}\left(\overline{\theta}\right) = \lambda(Z^e)\left[\int_1^1 \left[\left(P_i\left(\alpha_b, \overline{\varepsilon}_i\left(\overline{\theta}\right)\right)'_{\varepsilon} - \phi'_{\varepsilon}\left(\overline{\varepsilon}_i\left(\overline{\theta}\right)\right)\right)\frac{1}{x_i\sigma_\theta}\right]dx_i - \frac{1}{\overline{\theta}}\left[P_i(\alpha_b, 0) - \phi(0)\right]\right]$$

$$K'_R\left(\overline{\theta}\right) = -\lambda(Z^e)\frac{1}{\overline{\theta}}\left[P_i(\alpha,0) - \phi(0)\right] < 0 \text{ since } P_i(\alpha,0) > \phi(0).$$

For $R = R_c$ we have:

$$K'_R(R_c) = \lambda(Z^e)\left[\int_{\frac{R_c}{\overline{\theta}}}^1 \left[\left(P_i(\alpha, \overline{\varepsilon}_i(R_c))'_{\varepsilon} - \phi'_{\varepsilon}(\overline{\varepsilon}_i(R_c))\right)\frac{1}{x_i\sigma_\theta}\right]dx_i - \frac{1}{\overline{\theta}}\left[P_i(\alpha,0) - \phi(0)\right]\right]$$

and $K'_R(R_c) > 0$ if

$$\frac{1}{\sigma_\theta} > \frac{\frac{1}{\overline{\theta}}\left[P_i(\alpha,0) - \phi(0)\right]}{\int_{\frac{R_c}{\overline{\theta}}}^1 \left[\left(P_i(\alpha, \overline{\varepsilon}_i(R_c))'_{\varepsilon} - \phi'_{\varepsilon}(\overline{\varepsilon}_i(R_c))\right)\frac{1}{x_i}\right]dx_i}$$ which is true for σ_θ sufficiently small.

The proof of Lemma 2 is completed.

Proof of part a. of Proposition 1.

In order to prove the existence of a unique maximum for $V_b^e(R) = \frac{1}{\delta}\left[\int_{\overline{x}_i(R)}^1 \left[(1 - \phi(\overline{\varepsilon}_i(R)))R + \phi(\overline{\varepsilon}_i(R))Z^e - 1\right]dx_i + K(R)\right]$ we use Darboux's Theorem.[21]

21. Lars O., (2004). *A New Proof of Darboux's Theorem*. The American Mathematical Monthly, **111**(8), 713-715

$$V_b^e(R) = \frac{1}{\delta}\left[\int_{\overline{x}_i(R)}^{1}\left[(1-\phi(\overline{\varepsilon}_i(R)))R + \phi(\overline{\varepsilon}_i(R))Z^e - 1\right]dx_i + K(R)\right]$$
$$= \frac{1}{\delta}[B(R) + K(R)].$$

$V_b^e(R)$ is continuous and differentiable on $R \in [R_c, \overline{\theta}]$.

According to Lemma 1 and 2 we have:

$$\frac{\partial V_b^e(R)}{\partial R}(R_c) = \frac{1}{\delta}\left[B_R'(R_c) + K_R'(R_c)\right] > 0 \text{ as } B_R'(R_c) > 0$$

and $K_R'(R_c) > 0$.

$$\frac{\partial V_b^e(R)}{\partial R}(\overline{\theta}) = \frac{1}{\delta}\left[B_R'(\overline{\theta}) + K_R'(\overline{\theta})\right] < 0 \text{ as } B_R'(\overline{\theta}) < 0$$

and $K_R'(\overline{\theta}) < 0$.

The Darboux's Theorem conditions are fulfilled and we can conclude that there is a unique $R^* \in \left]R_c, \overline{\theta}\right[$ such that $V_b^e(R^*) = 0$ and R^* is a maximum for $V_b^e(R)$.

Moreover, since $R^* > R_c$ we have $B(R^*) > 0$, $K(R^*) > 0$ and $V_b^e(R^*) > 0$.

Part b. of proposition 1

Recall that $\overline{x}_i(R) = \frac{R}{\overline{\theta}}$ and it is obvious that $\frac{\partial \overline{x}_i(R)}{\partial R}(R^*) = \frac{1}{\overline{\theta}} > 0$.

Moreover, since the total level of financing is given by $D = \int_{\overline{x}_i(R)}^{1} 1\, dx_i$ $= \left(1 - \frac{R}{\overline{\theta}}\right)$ we have $\frac{\partial D(R)}{\partial R}(R^*) = D'(R^*) = -\frac{1}{\overline{\theta}} < 0$. Consequently, the total level of financing increases as the equilibrium rate of return charged by the bank decreases.

Part c. of proposition 1

The level of the equity multiplier is given by $\ell^*(R^*) = \frac{D^*(R^*)}{K^*(R^*)}$.

Consequently, we have $\frac{\partial \ell^*(R^*)}{\partial R^*} = \frac{D'(R^*)K(R^*) - D(R^*)K'(R^*)}{K^2(R^*)} < 0$

since $D'(R^*) < 0; K(R^*) > 0; D(R^*) > 0$ and $K'(R^*) > 0$ for σ_θ sufficiently small.

Proof of proposition 1 is completed.

Appendix 2. Proof of proposition 2.

Proof of part a.

The ex-post value of the bank is given by equation (15)

$$V_b(R) = \int_{x_c(R)}^{1} R dx_i + \int_{\overline{x}_i(R)}^{x_c(R)} \overline{Z} dx_i - \left[\int_{\overline{x}_i(R)}^{1} 1 dx_i - K_b(R) \right].$$

The bank goes into bankruptcy when

$$V_b(R) - K_r(R) < 0 \cdot$$

As $\hat{b} = K_b(R) - K_r(R)$, we have

$$V_b(R) - K_r(R) = \int_{x_c(R)}^{1} R dx_i + \int_{\overline{x}_i(R)}^{x_c(R)} \overline{Z} dx_i - \int_{\overline{x}_i(R)}^{1} dx_i + \hat{b}$$

$$V_b(R) - K_r(R) = R[1 - x_c(R)] + \overline{Z}[x_c(R) - \overline{x}_i(R)] - [1 - \overline{x}_i(R)] + \hat{b} < 0$$

Or equivalently with $x_c(R) = \dfrac{R}{\overline{\overline{\theta}} + \varepsilon_c \sigma_\theta}$ and $\overline{x}_i(R) = \dfrac{R}{\overline{\overline{\theta}}}$

$$R\left[1 - \frac{R}{\overline{\overline{\theta}} + \varepsilon_c \sigma_\theta}\right] + \overline{Z}\left[\frac{R}{\overline{\overline{\theta}} + \varepsilon_c \sigma_\theta} - \frac{R}{\overline{\overline{\theta}}}\right] - \left[1 - \frac{R}{\overline{\overline{\theta}}}\right] + \hat{b} < 0 .$$

Consequently, there is $\varepsilon_c = \dfrac{\overline{\theta}\left[(R-1)(R-\overline{\theta}) - \hat{b}\overline{\theta}\right]}{\sigma_\theta\left[R(1-\overline{Z}) + \overline{\theta}(R-1) + \hat{b}\overline{\theta}\right]}$ such that

$$R\left[1 - \frac{R}{\overline{\overline{\theta}} + \varepsilon_c \sigma_\theta}\right] + \overline{Z}\left[\frac{R}{\overline{\overline{\theta}} + \varepsilon_c \sigma_\theta} - \frac{R}{\overline{\overline{\theta}}}\right] - 1\left[1 - \frac{R}{\overline{\overline{\theta}}}\right] + \hat{b} = 0$$

and ε_c is the critical level of the macroeconomic shock at which the bank goes into bankruptcy.

Proof of part b.

We search for the value of R that minimises the bank's probability of default. We know that this probability of default decreases with the value of ε_c .

$$\frac{\partial \varepsilon_c}{\partial R} = \frac{\bar{\theta}\left[\left(\bar{Z} - 2R\bar{\theta}\right) + R^2\left(\bar{\theta} + 1 - \bar{Z}\right)\right]}{\sigma_\theta\left[R\left(1 - \bar{Z}\right) + \bar{\theta}\left(R - 1\right)\right]^2} = 0 \text{ for}$$

$$R_1 = \frac{\bar{\theta}\left(1 - \hat{b}\right) + \sqrt{\left(1 - \hat{b}\right)\bar{\theta}\left[\left(1 - \bar{Z}\right)\left(\theta - \bar{Z}\right) + \hat{b}\bar{\theta}\right]}}{1 + \bar{\theta} - \bar{Z}}$$

$$R_2 = \frac{\bar{\theta}\left(1 - \hat{b}\right) - \sqrt{\left(1 - \hat{b}\right)\bar{\theta}\left[\left(1 - \bar{Z}\right)\left(\theta - \bar{Z}\right) + \hat{b}\bar{\theta}\right]}}{1 + \bar{\theta} - \bar{Z}}$$

and $R_2 < 1 < R_1$ for $\hat{b} < \dfrac{\left(1 - \bar{Z}\right)\left(\bar{\theta} - 1\right)}{\left(2 - \bar{Z}\right)\bar{\theta}} < 1$.

As $\dfrac{\partial^2 \varepsilon_c}{\partial R^2} = \dfrac{\bar{\theta}\left[2\bar{\theta}\left(\bar{\theta} - R\right)\left(1 - \hat{b}\right) + R^2\left(1 - \bar{Z} + \bar{\theta}\right)\right]}{\sigma_\theta\left[R\left(1 - \bar{Z}\right) + \bar{\theta}\left(R - 1 + b\right)\right]^2} > 0$, there is a unique value

$$R_{\min} = \frac{\bar{\theta}\left(1 - \hat{b}\right) + \sqrt{\left(1 - \hat{b}\right)\bar{\theta}\left[\left(1 - \bar{Z}\right)\left(\theta - \bar{Z}\right) + \hat{b}\bar{\theta}\right]}}{1 + \bar{\theta} - \bar{Z}} > 1$$

that minimises the value of ε_c .

In addition, ε_c is decreasing between the two roots R_1, R_2 (which means that the bank's probability of default is also decreasing) whereas ε_c is increasing outside the two roots R_1, R_2 (which means that the bank's probability of default is also increasing).

Finally, there is a unique value of bank leverage $\ell\left(R_{\min}\right) = \dfrac{D\left(R_{\min}\right)}{K_b\left(R_{\min}\right)}$ that minimises the bank's probability of default and the relation between the bank's leverage and the bank's probability of default is nonlinear. In fact, the bank's probability of default is decreasing between $\left]1, R_{\min}\right[$ (between the two roots of the equation) and is increasing between $\left]R_{\min}, \bar{\theta}\right[$ (outside the two roots of the equation).

Appendix 3. Proof of proposition 3.

Proof of part a.

We have to show that $\ell(R^*(Z^e)) = \dfrac{D(R^*(Z^e))}{K_b(R^*(Z^e),Z^e)}$ is an increasing function of the expected value of the collateral Z^e. Taking the derivative of $\ell(R^*)$, we have

$$\frac{d\ell(R^*)}{dZ^e} = \frac{\dfrac{dD(.)}{dZ^e}K_b(.) - D(.)\dfrac{dK_b(.)}{dZ^e}}{K_b(.)^2}$$

with $\dfrac{dD(.)}{dZ^e}(R^*) = \dfrac{\partial D(.)}{\partial R}(R^*)\dfrac{\partial R(.)}{\partial Z^e}(R^*)$

and $\dfrac{dK_b(.)}{dZ^e}(R^*) = \dfrac{\partial K_b(.)}{\partial R}(R^*)\dfrac{\partial R(.)}{\partial Z^e}(R^*) + \dfrac{\partial K_b(.)}{\partial Z^e}$.

As $K_b(R^*,Z^e) > 0, D(R^*,Z^e) > 0$, $\dfrac{\partial D(.)}{\partial R}(R^*) < 0$, $\dfrac{\partial K_b(.)}{\partial Z^e} < 0$, $\dfrac{\partial K_b(.)}{\partial R}(R^*) > 0$, we are sure that $\dfrac{d\ell(R^*)}{dZ^e} > 0$ if $\dfrac{\partial R(.)}{\partial Z^e}(R^*) < 0$.

We have

$$V_b^e(R^*) = \frac{1}{(1+\delta)}\left[\int_{\bar{x}_i(R^*)}^{1}\left[(1-\phi(\bar{\varepsilon}_i(R^*)))R^* + \phi(\bar{\varepsilon}_i(R^*))Z^e - 1\right]dx_i + K_b(R^*)\right]$$

$$V_b^e(R^*) = \frac{1}{(1+\delta)}[B(R^*) + K_b(R^*)]$$

the maximum net expected value of the bank. Takes the total derivative of $V_b^e(R^*)$ and equates it with zero:

$$dV_b^e(R^*) = \frac{\partial V_b^e(R)}{\partial Z^e}(R^*)dZ^e + \frac{\partial V_b^e(R)}{\partial R}(R^*)dR^* = 0$$

and $\dfrac{dR^*}{dZ^e} = -\dfrac{\dfrac{\partial V_b^e(R)}{\partial Z^e}(R^*)}{\dfrac{\partial V_b^e(R)}{\partial R}(R^*)}$

We have:

$$\frac{\partial V_b^e(R)}{\partial Z^e}(R^*) = \frac{\partial B(R)}{\partial Z^e}(R^*) + \frac{\partial K_b(R)}{\partial Z^e}(R^*)\ \text{BB}$$

$$\frac{\partial V_b^e(R)}{\partial R}(R^*) = \frac{\partial B(R)}{\partial R}(R^*) + \frac{\partial K_b(R)}{\partial R}(R^*) = \frac{\partial K_b(R)}{\partial R}(R^*)$$

As $\frac{\partial K_b(R)}{\partial R}(R^*) > 0$ we have $\frac{\partial V_b^e(R)}{\partial R}(R^*) > 0$

$$\frac{\partial B(R)}{\partial Z^e}(R^*) = \int_{\overline{x}_i(R)}^{1} \phi(\overline{\varepsilon}_i(R))\, dx_i$$

and $\frac{\partial K_b(R)}{\partial Z^e}(R^*) = \frac{\partial \lambda(Z^e)}{\partial Z^e} \int_{\overline{x}_i(R)}^{1} (P_i(\alpha_b, \overline{\varepsilon}_i(R)) - \phi(\overline{\varepsilon}_i(R)))\, dx_i$.

Consequently,

$$\frac{dR^*}{dZ^e} = -\frac{\frac{\partial V_b^e(R)}{\partial Z^e}(R^*)}{\frac{\partial V_b^e(R)}{\partial R}(R^*)} < 0 \ \text{ if } \ \frac{\partial B(R)}{\partial Z^e}(R^*) \geq \frac{\partial K_b(R)}{\partial Z^e}(R^*) \ \text{ which}$$

is true for

$$\frac{\partial \lambda(Z^e)}{\partial Z^e} \leq \frac{\int_{\overline{x}_i(R)}^{1} \phi(\overline{\varepsilon}_i(R))\, dx_i}{\int_{\overline{x}_i(R)}^{1} (P_i(\alpha_b, \overline{\varepsilon}_i(R)) - \phi(\overline{\varepsilon}_i(R)))\, dx_i}$$

This means that when the expected value of the collateral increases, the equilibrium rate of return charged by the bank decreases and the value of leverage increases.

Proof of part b.

We search for the critical value Z_c^e for which $\ell(R^*) > \ell(R_{\min})$

According to proposition 1, there is an equilibrium rate of return for the bank if $\overline{\theta} > 2 - Z^e$ or, put differently, for $Z^e > 2 - \overline{\theta} < 1$ with $\overline{\theta} > 1$.

Thus, for $Z^e < 2 - \overline{\theta} < 1$ there is no equilibrium and $\ell(R^*) = 0$.

Also, since $K_b(R) = \int_{\overline{x}_i(R)}^{1} \lambda(Z^e)(P_i(\alpha_b, \overline{\varepsilon}_i) - \phi(\overline{\varepsilon}_i))dx_i$

and $\underbrace{\lim}_{Z^e \to 1} \lambda(Z^e) \to 0$, we have

$$\underbrace{\lim}_{Z^e \to 1} \ell(R^*) \to +\infty .$$

Thus, $\ell(R^*)$ is an increasing function of Z^e, $\forall Z^e \in]0,1[$.

Since $\ell(R_{\min}) = \dfrac{D(R_{\min}, \overline{Z})}{K_b(R_{\min}, \overline{Z})}$ with

$$R_{\min} = \frac{\overline{\theta}\left(1-\hat{b}\right) + \sqrt{\left(1-\hat{b}\right)\overline{\theta}\left[\left(1-\overline{Z}\right)\left(\theta - \overline{Z}\right) + \hat{b}\overline{\theta}\right]}}{1 + \overline{\theta} - \overline{Z}}$$

and $\overline{Z} < 1$ we have $0 < \ell(R_{\min}) < \infty$.

Consequently, there is $Z_c^e \in [0;1]$ such that $\ell(R^*(Z_c^e)) = \ell(R_{\min})$ and $\ell(R^*(Z^e)) > \ell(R_{\min})$ for $Z^e > Z_c^e$.

The proof of proposition 3 is completed.

Appendix 4. Minimum fragility leverage value and "area of increasing financial fragility" in the case of "bad business climate".

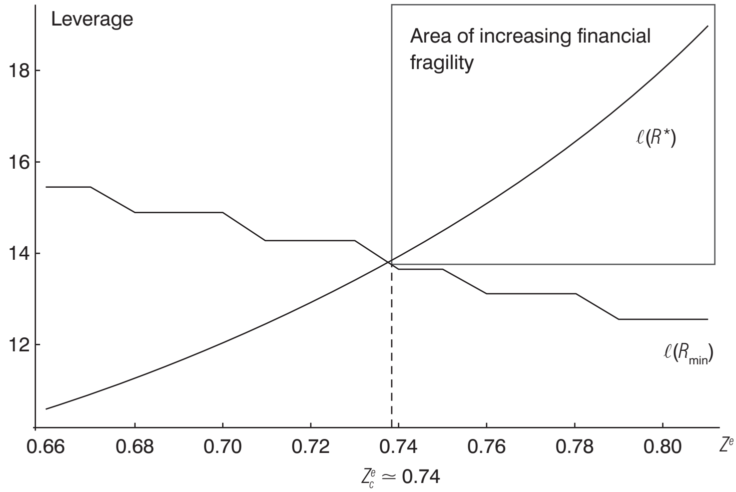
The set of parameters is chosen in order to illustrate an economic situation corresponding to a "bad business climate". Consequently, the sensitivity of the productivity to the macroeconomic shock (σ_θ) is higher than before whereas the maximum rate of return on financed projects $\left(\overline{\theta}\right)$ is lower. However, for comparison purpose, we retain the same discounted rate of profit and debt recovery rate $\left(\overline{Z}\right)$ (5% and 55% respectively).

Table 2. Values of the parameters for a "bad business climate"

$\overline{\theta}$	σ_θ	$\overline{Z}$	$(1+\delta)$
1.45	0.08	0.55	1.05

Figure 5 clearly shows that the size of the "area of increasing financial fragility" is lower than in the situation of good business climate as the critical expected value of the collateral is now about 0.74 compared to 0.72

Figure 5. Level of leverage and area of increasing financial fragility for "bad business climate"



in case of good business climate. This result may seem paradoxical since the "area of increasing financial fragility" begins for value of bank leverage lower than in case of good business climate. It is explained by the fact that, for each expected value of the collateral, the equilibrium level of leverage chosen by the bank is lower in case of bad business climate than in case of good business climate.